# Bequeathing in ambiguous times

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# Bequeathing in Ambiguous Times

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**Abstract**

This paper studies a model of altruistic bequest with ambiguity. The parent is under ambiguity about the future economic condition, which affects the probability of the child's economic success. I show that an increase in the degree of ambiguity makes the parent leave more transfer. Our model implies that the amount of parental transfers grows during a pandemic or recession.

Keywords: Bequest; Parental Transfer; Robust Control; Ambiguity

JEL Classification: D13; D15; D81; E21

Highlights:

- An altruistic bequest model with ambiguity (or Knightian uncertainty) is studied.
- With ambiguity, the parent leaves a higher amount of bequest which induces the child to exert less effort.

## 1 Introduction

The ongoing pandemic is making people worry about their wealth as well as their health. Due to the drastic rise of fatality rate, people are being confronted the possibility of their own death. As a consequence, people are incentivised to prepare for their bequest, and the bequeathing decisions are been taken during the pandemic.[1]

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[1]Several articles have already pointed out the phenomenons related to this speculation. Huges (2020), for example, reports that legal firms in the UK have been receiving more enquiries about writing wills and estate planning than before the Covid-19 pandemic hit.

The question this paper address is, whether a parent leaves more bequest during a pandemic. During a pandemic, decisions are taken under an extreme ambiguity about the future. People cannot properly estimate the probabilities of the future economic conditions, and thus decisions are hugely depend on the individuals' subjective expectation. To analyse economic activities in this situation, supposing the rational expectation—which is a workhorse assumption in economics—is inadequate.

Taking the discussion as a motivation, this paper studies a model of altruistic bequest where a parent lacks confident about the future economic conditions which is relevant with the child's economic success. We do not explicitly consider the elements of a pandemic in our model, rather we build a general model of bequest with ambiguity, and interpret a rise in the degree of ambiguity as a pandemic shock or a recession shock. Following the multiplier preferences of Hansen and Sargent (2001), the parent endogenously tilts the subjective expectation, and the ambiguity-aversion is endogenously formed. To the best of our knowledge, this paper is the first attempt to study bequeathing decisions with ambiguity.[2]

We show that more level of ambiguity increases the amount of bequest that the parent leaves. In equilibrium, the parent equates the marginal utility from his/her own consumption and the subjective expected utility of the child's future consumption. Due to the strong fear about the negative condition in the future, the subjective expected utility is lower than the one rationally expected. As a result, the parent leaves mode bequest as the degree of ambiguity increases. Since the bequest dis-incentivises the child's effort, the result implies that the increased parent's ambiguity decreases the child's effort. The results cannot be described by the traditional bequest theory which only considers the risks and uncertainty but not ambiguity.

Most of the literature of the parental transfers is based on the altruistic model (e.g. Becker 1974; Becker and Tomes 1979; Becker 1981) in which parents bequeath to gain utility from the increase of the child's utility. The exchange models, such as Bernheim et al. (1985) and Cox (1987), extend the altruistic model to add the parent's strategic exchange motive which values the children's consumption of particular goods or activities. While the literature investigates several other bequest motives and its consequence, to my knowledge, none has studied the bequeathing decisions without parents' rational expectations.[3]

Although the rational expectation is a common assumption in economics, it is well studied

[2]If there exists risk, the probability function is known and decisions are taken with it. With ambiguity (or Knightian uncertainty), the true probability function is vague and there exists a set of possibly-probability functions. The literature of parental transfers has focused on the decisions by risk-averse parents, whereas this paper considers a risk-averse and ambiguity-averse parent.

[3]Another large branch of the literature studies the consequences of accidental bequest such as Yaari (1965) and Davies (1981).

that several experimental and empirical evidences do not support it. (e.g. Ellsberg 1961; Mehra and Prescott 1985). Because of the discrepancy between the theory and evidences, there is a growing number of literature which studies models with ambiguity aversion. Most of this literature is in the fields of asset pricing (e.g. Epstein and Wang 1994; Barillas et al. 2009), monetary policymaking (e.g. Walsh 2004; Woodford 2010; Dennis 2010) and fiscal policymaking. (e.g. Svec 2012; Karantounias 2013 ; Ferriere and Karantounias 2019)[4]

# 2 Model and Results

We consider a modification of the parent-child model of Becker (1974). There is a household consists of an altruistic parent and a selfish child. The child's income $y_k$ is $y_h$ with probability $p$, and is $y_l$ with probability $1-p$, where $y_l < y_h$; The probability $p$ is a function of effort and a random variable $\epsilon$. For simplicity, this paper focuses on the case where the child know the following true distribution:

$$p = p(e, \epsilon) \coloneqq e + \epsilon \tag{1}$$

Let $\mu(\epsilon)$ be the probability measure of $\epsilon$ and $\int_{\underline{\epsilon}}^{\bar{\epsilon}} \mu(\epsilon) d\epsilon = 1$ where $\bar{\epsilon} \in \mathbb{R}_+$ and $\underline{\epsilon} \in \mathbb{R}_-$ be the maximum and minimum values of $\epsilon$, respectively.[5]

The timing of events is as follows:

1. the parent consumes $c_p$ and leaves monetary transfer (bequest; $b$) to the child;
2. nature chooses $\epsilon$;
3. the child choose work effort $e$; and
4. $y_k = \{y_h, y_l\}$ is determined by nature. The child choose consumption $c_k$.

The only difference between the traditional model and our model is that we have $\epsilon$ and the parent is not perfectly sure about its probability function. We solve the model by backward induction.

## 2.1 Child's Decision

Given the bequest $b$, child chooses $e$ that maximises the expected utility:

$$\mathbb{E}\left[U_k(c_k, e)\right] = pu(c_k) + (1-p)u(c_k) - v(e) \tag{2}$$

[4]SeeHansen and Sargent (2008) for more examples of applications.

[5]Note that if $\bar{\epsilon} = \underline{\epsilon} = 0$, the model coincides with the standard altruistic parent model.

subject to the budget constraint:

$$c_k = y_k + b \tag{3}$$

where $u$ is the utility from consumption and $-\upsilon$ represents the dis-utility of effort. Both functions are continuously-differentiable, $u$ is strictly concave ($u' > 0$, $u'' < 0$), and $\upsilon$ is strictly convex ($\upsilon' > 0$ , $\upsilon'' > 0$).

Denote $u_i = u(y_i + b)$. The first order condition is written as:

$$\frac{d\mathbb{E}\left[U_k(e)\right]}{de} = (u_h - u_l)\, p'(e,\epsilon) - \upsilon'(e) = 0 \tag{4}$$

which yields the optimal level of effort $e^* = e^*(b,\epsilon)$. The equation shows that $e^*$ makes expected marginal utility of consumption equals to the marginal dis-utility of exerting effort. From Equation (4), we have:

$$\frac{de^*}{db} = \frac{(u_h^{'} - u_l^{'})p'(e,\epsilon)}{(u_h - u_l)p''(e,\epsilon) + \upsilon''(e)} < 0 \tag{5}$$

which illustrates that the effort is dis-incentivised by the bequest.

## 2.2 Bequeathing without Ambiguity

As a benchmark, we briefly analyse an economy where the parent knows the true probability measure $p(e,\epsilon)$. Given the budget constraint: $c_p + b = y_p$ and the child's decision function $e^*(b,\epsilon)$, the parent selects the amount of bequest that maximises the rationally expected utility:

$$\mathbb{E}\left[U_p\left(b\right)\right] = u_p + \beta \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left(pu_h + (1-p)u_l\right) \mu(\epsilon) d\epsilon \tag{6}$$

where $u_p = u(y_p - b)$, $\beta$ is the intercohort discount factor (or degree of altruism) such that $0 < \beta < 1$. The first order condition is:

$$-u_p^{'} + \beta \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left(pu_h^{'} + (1-p)u_l^{'}\right) \mu(\epsilon) d\epsilon = 0 \tag{7}$$

which yields $b^*$—the amount of bequest the parent leave if there is no ambiguity. Equation (7) balances the marginal utility of decreasing the parent's consumption with the *rationally* expected marginal utility of increasing the child's consumption.

## 2.3 Bequeathing under Ambiguity

Suppose now that the parent have a limited degree of ambiguity about the true distribution of the random variable $\epsilon$, and the parent consider the true distribution is in a set of possible alternative distributions. Following Hansen and Sargent (2001), we apply the robust control theory to solve this problem.

Let $\tilde{\mathbb{E}}$ be the parent's subjective expectation of a random variable $x$ and suppose the absolute continuity with respect to $p$. Then the Radon-Nikodym theorem indicates that there exists a measurable function $m$ such that $\tilde{\mathbb{E}}[x] = \mathbb{E}[mx]$ where $\mathbb{E}[m] = 1$ is supposed. Following the literature of robust control, we measure the distance between the actual and approximating models by the relative entropy: $\mathbb{E}\left[m \ln m\right]$ which is convex and grounded.

Let $V_k(b, \epsilon)$ be child's the value function: $V_k(b, \epsilon) = U_k(e^*(b, \epsilon))$. Given $e^*(b, \epsilon)$, the parent choses $b$ which maximises:

$$\tilde{\mathbb{E}}\left[U_p\left(b\right)\right] = u_p + \min_m \left\{ \beta \int_{\underline{\epsilon}}^{\bar{\epsilon}} m(\epsilon) \left(V_k(b, \epsilon) + \theta \ln m(\epsilon)\right) \mu(\epsilon) d\epsilon - \beta \theta \lambda \left( \int_{\underline{\epsilon}}^{\bar{\epsilon}} m(\epsilon) \mu(\epsilon) d\epsilon - 1 \right) \right\}$$

where $\lambda$ is the Lagrangian multiplier of the legitimate constraint: $\int_{\underline{\epsilon}}^{\bar{\epsilon}} m(\epsilon)\mu(\epsilon)d\epsilon = 1$ which ensures each approximating model is a legitimate probability model. $\theta$ is a penalty parameter which assess the parent's degree of ambiguity about the probability measure. A higher $\theta$ implies that the parent is more confident to the approximating model. As $\theta \to \infty$, the parent has a full confidence to the approximating model, and it coincides with the rational expectation model studied in the previous section.

Using the envelop theorem, the first order condition of the inner minimisation problem is given by:

$$\beta \left(V_k(b, \epsilon) + \theta \left(1 + \ln m(\epsilon)\right) - \lambda \theta\right) = 0 \tag{8}$$

With the legitimate constraint and Equation (7), we have the optimal distortion $\tilde{m}(\epsilon)$ such that:

$$\tilde{m}(\epsilon) = \frac{\exp\left(-\frac{V_k(b,\epsilon)}{\theta}\right)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} \exp\left(-\frac{V_k(b,\epsilon)}{\theta}\right) \mu(\epsilon) d\epsilon}. \tag{9}$$

The optimal distortion puts a higher probability on bad scenario (low realisation of $\epsilon$) than the actual probability. The size of the optimal distortion increases as the penalty parameter $\theta$ decreases, and vice versa.[6]

With $\tilde{m}(\epsilon)$, the parent's objective function is written as:

[6] Note that the optimal distortion converges to 1 as $\theta \to \infty$ for all $\epsilon$.

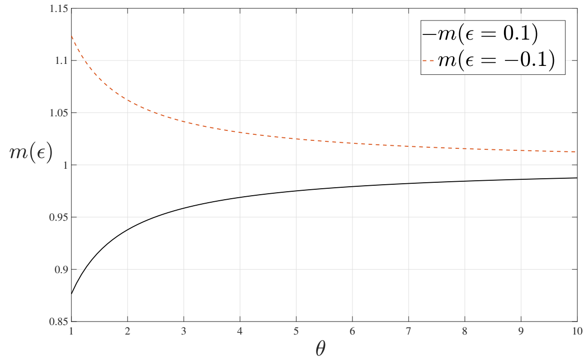



Figure 1: Optimal Distortion
Note: The functional forms and the parameter values follow Assumption 1.

$$\tilde{\mathbb{E}}\left[U_p\left(b\right)\right] = u_p(y_p - b) - \beta\theta \ln\left(\int_{\underline{\epsilon}}^{\overline{\epsilon}} \exp\left(-\frac{V_k(b,\epsilon)}{\theta}\right)\mu(\epsilon)d\epsilon\right). \tag{10}$$

Using the envelop theorem, the first order condition is:

$$\frac{d\tilde{\mathbb{E}}\left[U_p(b)\right]}{db} = -u_p^{'}(y_p - b) + \beta \int_{\underline{\epsilon}}^{\overline{\epsilon}} \tilde{m}(\theta) \underbrace{\left(pu_h^{'} + (1-p)u_l^{'}\right)}_{V_k'} \mu(\epsilon)d\epsilon = 0, \tag{11}$$

which yields $\tilde{b}$—the amount of bequest left by the parent who faces the ambiguity. The equation balance the marginal utility of decreasing the parent's consumption with the *subjectively* expected marginal utility of increasing the child's consumption.

**Proposition 1**

1. *More ambiguity induces the parent to leave bequest more:*

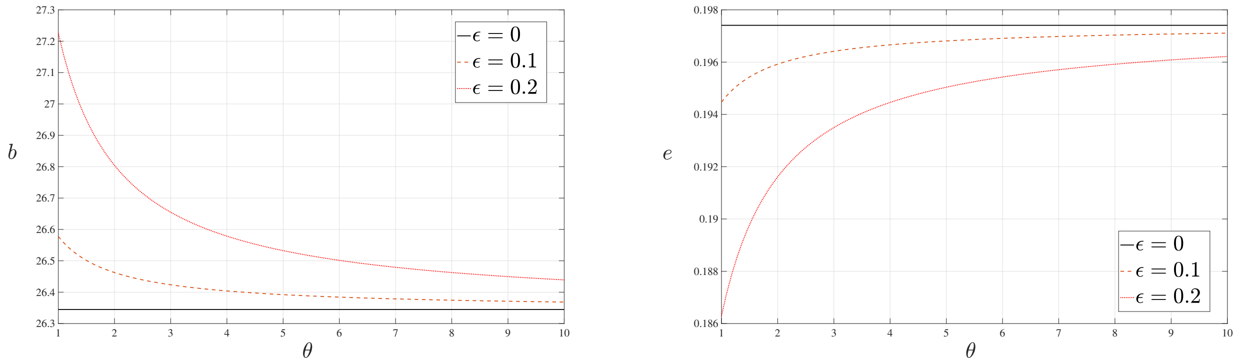
$$-\frac{d\tilde{b}}{d\theta} > 0.$$

2. *More ambiguity decreases the effort the child exerts:*

$$\frac{de^*}{d\theta} < 0.$$

3. *Comparing to the results with the rational expectation scenario, we have:*

$$b^* \leq \tilde{b},$$



(a) Bequest (b) Effort

Figure 2: Comparative Statistics
Note: The functional forms and the parameter values follow Assumption 1.

$$e(\tilde{b}) \leq e(b^*),$$

*where the equalities hold as $\theta \to \infty$.*

***Proof.***

*Since* $\frac{d\left(\frac{d\tilde{\mathbb{E}}[U_p(b)]}{db}\right)}{d\theta} = \frac{d\left(\beta\int_{\underline{\epsilon}}^{\overline{\epsilon}}\tilde{m}(\theta)\left(pu_h'+(1-p)u_l'\right)\mu(\epsilon)d\epsilon\right)}{d\theta} = \beta\int_{\underline{\epsilon}}^{\overline{\epsilon}}\frac{d\tilde{m}(\theta)}{d\theta}\left(pu_h'+(1-p)u_l'\right)\mu(\epsilon)d\epsilon > 0$, *Eq.11 implies the first result. The second result follows from the first result and Eq.5. The third result follows from the two results and the property that* $\lim_{\theta\to\infty}\tilde{m}(\epsilon) = 1$.

In the face of ambiguity, the parent leaves a higher amount of bequest than the time without the ambiguity, due to the strong fear about the negative economic circumstances. Since the bequest dis-incentivises the child's effort (Equation 5), the parent's ambiguity has a negative effect on the child's effort choice.

**Assumption 1.** (Numerical Example) *The functions are supposed as $u(c) = \ln c$ and $-\upsilon(e) = \ln(1-e)$. The shock takes $\epsilon$ or $-\epsilon$ with probability $q$ and $1-q$, respectively. The parameters are: $\beta = 0.985$, $y_h = 1.5$, $y_l = 0.5$, and $y_p = 1$.*

The effect of ambiguity on the results is illustrated in Figure 2. The solid line in each figure depicts the outcome corresponding to $\epsilon = 0$ implying there is no source of ambiguity, as in the traditional altruistic bequest models.[7] In this case, the lines are horizontal in the sense that the penalty parameter $\theta$ does not affect the outcome. For all $\theta$, the outcomes completely coincide with the ones in the rational expectation scenario studied in Section 2-1.

[7]Note that even in the case with $\epsilon = 0$, the uncertainty about the success is still exist.

The dotted lines depict the results when $\epsilon = 0.1$ and $\epsilon = 0.2$. They show the mechanism which cannot be described by the traditional theory with uncertainty but without ambiguity—the parent leaves a higher amount of bequest as $\theta$ and (or) $\epsilon$ increase(s). Since the effort depends on the amount of bequest, the child exerts more effort as $\theta$ and(or) $\epsilon$ increase(s). As $\theta \to \infty$, both of the dotted lines converge to the solid line in both figures. Since $\theta \to \infty$ implies the parent's rational expectation, we can see that the value of $\epsilon$ does not affect the rational expectation model.

# 3 Concluding Remark

We have studied an altruistic bequest model with ambiguity by applying the robust control theory. We show that the parent leaves a larger amount of bequest than the case with the rational expectation.

The result suggests that when the future economic condition is volatile, ambiguity-averse parents leave more monetary transfer to their children.[8] As the degree of ambiguity increases, the amount of bequest increases while the child's effort decreases. This prediction is in contrast with the standard transfer models supposing the rational expectation, in which the parent does not change transfers even if the value of $\epsilon$ changes.

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[8] Another possible interpretation of the result is that the ambiguity related with the parent's unfamiliarity with the child's job. Then the probability of the child's success might be ambiguous for the parent, and the amount of bequest might be high. If the child is doing the same job as the parent, on the other hand, the ambiguity must be relatively small and the parent leaves the bequest as in the rational expectation model. The interpretation suggests that the child exerts higher effort if the parent is familiar with the job, and vice versa.

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